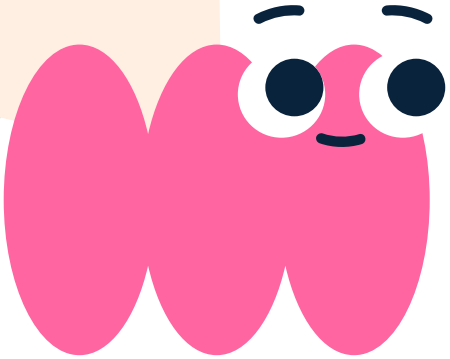
ALGEBRA



Prepared by
Scholarly Test Prep
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Algebra

Algebra is a branch of mathematics that involves the use of variables with mathematical operations.

Algebraic Expression

- **a combination of variables and numbers combined through the mathematical operations.**

3x + 2yz

This is an expression.

Variable

- **a letter or object that represents an unknown number or quantity.**
- **also called <u>pronumeral</u>.**

In this example,

3x + 2yz

the variables are **x**, **y**, and **z**.

Term

- **a part of an expression with only variables, numbers, multiplication, and division.**
- **It is separated by the addition and subtraction operations.**
- **A term with no variable is called a <u>constant term</u>.**

From the example above, **3x** and **2yz** are the terms.

Coefficient

- **The number in front of a variable.**
- **If no number is seen in front of a variable, its coefficient is 1.**
- **If a term subtracts, its coefficient is a negative number.**

From the example above, **3** and **2** are the coefficients.

Anatomy of an Algebraic Expression

An algebraic expression consists of one or more terms. Let's take a closer look at its components.

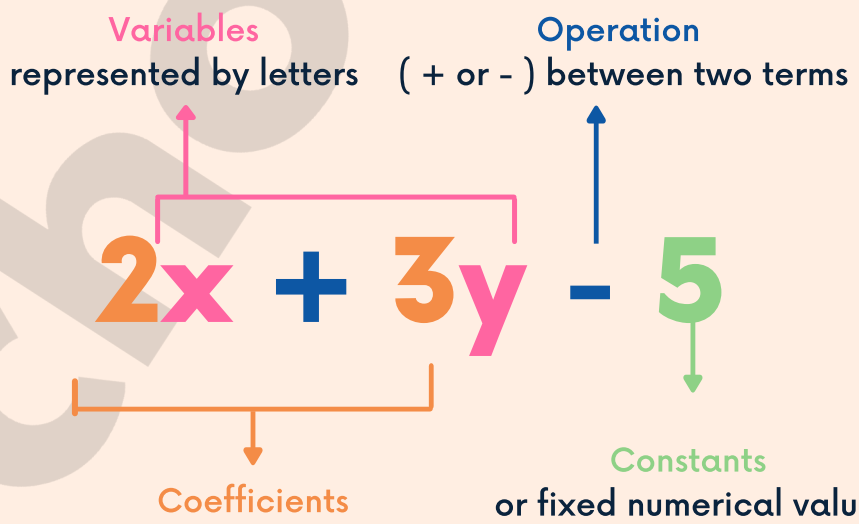
A term can be any of the following:

$-y$
A variable
a.k.a. pronumeral

3
A constant

$5x$
A variable and a constant combined by × or ÷

An Algebraic Expression may have following parts:



Operation Rules

1. a x b is written as ab.
2. a ÷ b is written as ab.
3. The sum of a and b is a + b.
4. The difference between a and b is a - b.
5. The product of a and b is ab.
6. The quotient of a and b is $\frac{a}{b}$.
7. The square of a is a^2.

Substitution

- replacing variables with numbers
- sometimes called <u>evaluation</u>

Equivalence

- If two different expressions have the same result after an equal substitution, they are equivalent.

Arithmetic Laws

Commutative Law

- If we add or multiply two numbers, then the resultant value remains the same, regardless of their order.

$$a + b = b + a$$
$$ab = ba$$

Associative Law

- The sum or product of three numbers is still the same regardless of the groupings of the numbers.

$$(a + b) + c = a + (b + c)$$
$$(ab) \times c = a \times (bc)$$

Distributive Law

- Multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

$$a(b + c) = ab + ac$$

Algebra Operations

Addition and Subtraction

In adding and subtracting an expression, the terms must be alike. Like terms are terms that consist of the same coefficient, variables, and powers, regardless of their order.

For example, $5xy^2$ and $5y^2x$ are like terms.

Example: Simplify 4x + 3y - 2x + 2y.

Step 1: Group like terms

$4x - 2x + 3y + 2y$

Step 2: Simplify.

$4x - 2x + 3y + 2y$
$= \mathbf{2x + 5y}$

Multiplication

Unlike addition and subtraction, multiplication does not require having like terms. Directly multiply the factors

For example, $2ab \times 5c$

$2 \times a \times b \times 5 \times c = 10abc$

Division

The division of expressions is expressed as fractions. Cancel out common factors and reduce the coefficients to their lowest terms.

Example: $12x^2yz \div 4xz$

$$\frac{12x^2yz}{4xz} = \frac{\cancel{12}^{3}x^{\cancel{2}^{1}}yz}{\cancel{4}xz} = 3xy$$

Algebra Fraction Operations

Addition and Subtraction

Like the rules in adding and subtracting normal unlike fractions, algebraic fractions must have the same denominators to proceed.

Example: $\frac{2a}{3} + \frac{5}{b}$

Step 1: **Find their LCD.**

In the example, their LCD is 3b. It will be their new denominator.

Step 2: **Divide and Multiply, then Simplify**

Divide the LCD by the denominators of the fractions and multiply the quotient by their numerators.

$$\frac{3b \div 3 \times 2a}{3b} + \frac{3b \div 3 \times 5}{3b} = \frac{2ab}{3ab} + \frac{15}{3ab}$$

$$= \frac{2ab + 15}{3b}$$

Therefore, the sum of $\frac{2a}{3}$ and $\frac{5}{b}$ is $\frac{2ab + 15}{3b}$.

Multiplication

The rules of multiplying fractions can also be applied to algebraic fractions. Directly multiply the numerators and denominators separately. Then, simplify their product.

Division

The rules of dividing fractions can also be applied to algebraic fractions. Transform the divisor into its reciprocal (interchanging the values of numerator and denominator) and multiply the fractions. Lastly, simplify the result.

Factoring Expressions

It is getting the factors of expressions to easily simplify them.

Example: Factorise 12xy + 3x

Step 1: **Find their Highest Common Factor (HCF).**

In our example, the HCF is 3x.

Step 2: **Divide the terms by the HCF.**

$$\frac{12xy + 3}{3x} = 4y + 1$$

Step 3: **Put the quotient and HCF together.**

Separate them with parenthesis.

$$12xy + 3x = 3x\,(4y + 1)$$